PARISHRAM 2024

- FOR 12TH CLASS CBSE BOARD STUDENTS -

- Subject Name- Mathematics
- Chapter Name- Relations and Functions

Lecture No.- 04

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Today's Targets

1. PYQs
2. Doubts
3. ---------
4. ---------

Homework

Let R be a relation over the set $N \times N$ and it is defined by $(a, b)R(c, d) \Rightarrow a + d = b + c$. Then R is

- **A** Symmetric only ✗
- **B** Transitive only ✗
- **C** Reflexive only ✗
- **D** None of these ✓

Definition

$(a,b)\,R\,(c,d)$

$\Rightarrow a+d = b+c$

① Reflexive ✓

$(a,b)\,R\,(a,b)$

$\Rightarrow a+b = b+a$ (Yes)

② Symmetry ✓

$(a,b)R(c,d)$

$\Rightarrow a+d=b+c$

$\Rightarrow b+c=a+d$

$\Rightarrow c+b=d+a$

$\Rightarrow (c,d)R(a,b)$

③ Transitive ✓

$(a,b)R(c,d)$ and $(c,d)R(e,f)$

$\Rightarrow a+d=b+c$, $c+f=d+e$

$\Rightarrow a-b=c-d$, $c-d=e-f$

$\Rightarrow a-b=e-f$

$\Rightarrow a+f=b+e$

$\Rightarrow (a,b)R(e,f)$

Question

$R: A \to A$

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $\{(a, b): |a - b| \text{ is divisible by } 4\}$, is an equivalence relation. Write all the equivalence classes of R. (Delhi 2015)

① Reflexive ✓

$|a-a| = 0 = 4(0)$

$\Rightarrow (a,a) \in R$

② Symmetry ✓

If $(a,b) \in R$

$\Rightarrow |a-b| = 4k$

$\Rightarrow |-(b-a)| = 4k$

$\Rightarrow |b-a| = 4k$

$\Rightarrow (b,a) \in R$

③ Transitive ✓

If $(a,b) \in R$, $(b,c) \in R$

$\Rightarrow |a-b| = 4k \quad |b-c| = 4p$

$\Rightarrow (a-b) = \pm 4k, (b-c) = \pm 4p$

$\Rightarrow (a-b)+(b-c) = \pm 4k \pm 4p$

$\Rightarrow (a-c) = \pm 4(k+p)$

$\Rightarrow |a-c| = 4\lambda \quad (\text{where } \lambda = k+p)$

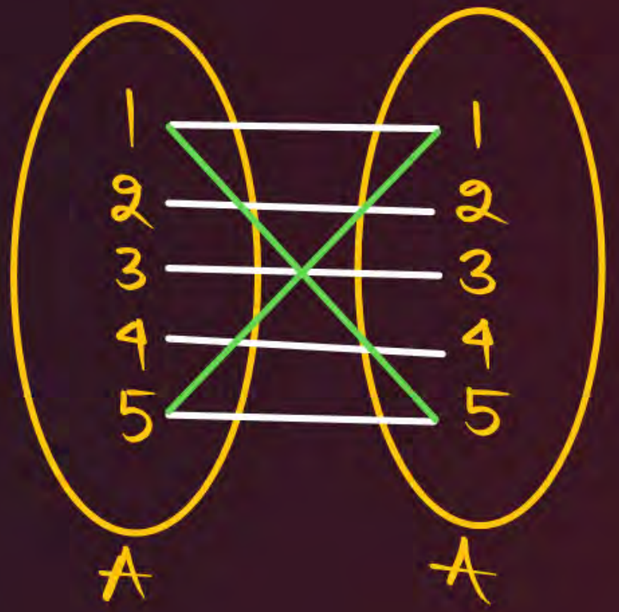
$\Rightarrow (a,c) \in R$

Equivalence Class

$[1] = [5] = \{1, 5\}$

or

$|a-b| = 4m$



Question

If N denotes the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation. (Delhi 2015)

③ Trans. ✓

If $(a,b)R(c,d)$, $(c,d)R(e,f)$

$\Rightarrow ad(b+c)=bc(a+d)$, $cf(d+e)=de(c+f)$

$\Rightarrow abd+acd=abc+bcd$, $cfd+cfe=ced+def$

$\Rightarrow cd(a-b)=ab(c-d)$, $cd(f-e)=ef(d-c)$

$\Rightarrow \frac{cd}{c-d}=\frac{ab}{a-b}$, $\frac{cd}{c-d}=\frac{-ef}{f-e}$

$\Rightarrow \frac{ab}{a-b}=\frac{-ef}{f-e}$

$\Rightarrow abf-abe=-aef+bef$

$\Rightarrow af(b+e)=be(a+f)$

$\Rightarrow (a,b)R(e,f)$

② Symm. ✓

If $(a,b)R(c,d)$

$\Rightarrow ad(b+c)=bc(a+d)$

$\Rightarrow bc(a+d)=ad(b+c)$

$\Rightarrow cb(a+d)=ad(b+c)$

$\Rightarrow (c,d)R(a,b)$

① Ref. ✓

$(a,b)R(a,b)$

$ab(b+a)=ba(a+b)$

$ab^2+ba^2=ba^2+ab^2$

(Yes)

Question

HW

$R: Z \to Z$

Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation. (CBSE 2018)

Question

$R : N \to N$

If R is a relation defined on the set of natural numbers N as follows: $R = \{(x, y) : x \in N, y \in N \text{ and } 2x + y = 24\}$, then find the domain and range of the relation R . Also, find whether R is an equivalence relation or not. (Delhi 2014)

a) $R = \{(x,y) : x, y \in N, 2x+y=24\}$

x	y	R
1	$22 \in N$	(1, 22)
2	$20 \in N$	(2, 20)
3	$18 \in N$	(3, 18)
4	$16 \in N$	(4, 16)
5	$14 \in N$	(5, 14)
6	$12 \in N$	(6, 12)
7	$10 \in N$	(7, 10)
8	$8 \in N$	(8, 8)
9	$6 \in N$	(9, 6)
10	$4 \in N$	(10, 4)
11	$2 \in N$	(11, 2)
12	$0 \notin N$	

Domain

$D_R = \{1,2,3,4,5,6,7,8,9,10,11\}$

Range

$R_R = \{2,4,6,8,10,12,14,16,18,20,22\}$

Codomain = N

b) ① Reflexive ✗

$(1,1) \notin R$

② Symm. ✗

$(1,22) \in R$

but $(22,1) \notin R$

③ Trans. ✗

$(7,10) \in R \quad (10,4) \in R$

but $(7,4) \notin R$

It is not equivalence relation

Question

Real No.s

chota ya barabar.

Show that the relation R on IR defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric. (Delhi 2019)

③ Trans. ✓

If $(a,b) \in R$, $(b,c) \in R$

$\Rightarrow a \leq b$, $b \leq c$

$\Rightarrow a \leq b \leq c$

$\Rightarrow a \leq c$

$\Rightarrow (a,c) \in R$

② Symm. ✗

If $(1,10) \in R$

$\Rightarrow 1 \leq 10$

but $10 \leq 1$ (false)

$\Rightarrow (10,1) \notin R$

① Ref. ✓

$a \leq a$? (Yes)

Question

$R : A \to A$

Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive. (All India 2019)

$R = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$

Rules: $a, b \in A$, $b = a + 1$

a	b	R
1	$2 \in A$	(1,2)
2	$3 \in A$	(2,3)
3	$4 \in A$	(3,4)
4	$5 \in A$	(4,5)
5	$6 \in A$	(5,6)
6	$7 \notin A$	

① Ref. ✗

$(1,1) \notin R$

② Symm. ✗

$(1,2) \in R$ but $(2,1) \notin R$

③ Trans. ✗

$(1,2), (2,3) \in R$

but $(1,3) \notin R$

Question

$R : A \to A$ $\quad A =$ set of all polygons

Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3,4 and 5?

① Refl. ✓

P_1 and P_1 have same no. of sides

$\Rightarrow (P_1, P_1) \in R$

② symm. ✓

If $(P_1, P_2) \in R$

$\Rightarrow P_1$ and P_2 have no. of sides

$\Rightarrow P_2$ and P_1 will also have same no. of sides

$\Rightarrow (P_2, P_1) \in R$

③ Trans. ✓

If $(P_1, P_2) \in R$, $(P_2, P_3) \in R$

$\Rightarrow P_1$ and P_2 have same no. of sides

$\Rightarrow P_2$ and P_3 have same no. of sides

$\Rightarrow P_1$ and P_3 have same no. of sides

$\Rightarrow (P_1, P_3) \in R$

Each element of set of all triangles are related to the T

Question

$R : A \to A$

Let R be the relation defined in the set A = {1,2,3,4,5,6,7} by R = {(a, b) : both a and b are either odd or even }. Show that R is an equivalence relation. Further, show that all the elements of the subset {1,3,5,7} are related to each other and all the elements of the subset {2,4,6} are related to each other, but no element of the subset {1,3,5,7} is related to any element of the subset {2,4,6}.

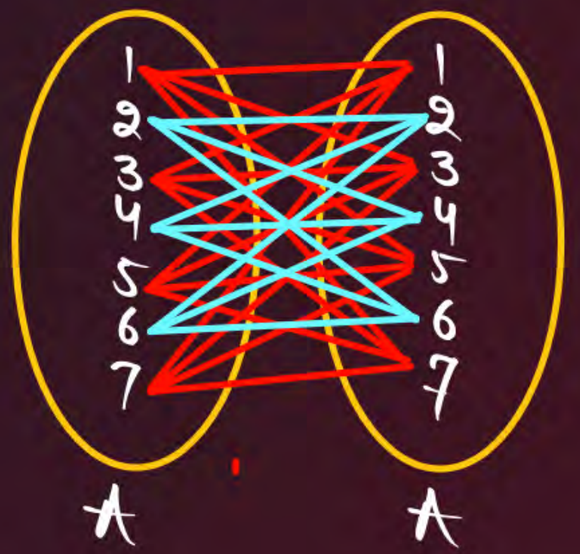
Sol:

$$R = \{(1,1),(1,3),(1,5),(1,7),(3,1),(3,3),(3,5),(3,7),(5,1),(5,3),(5,5),(5,7),(7,1),(7,3),(7,5),(7,7),(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}$$

Equivalence Class

$[1] = [3] = [5] = [7] = \{1,3,5,7\}$ ✓

$[2] = [4] = [6] = \{2,4,6\}$ ✓



Homework

(Competetive)

Let X = set of real numbers & $R = \{(x, y) : x - y + \sqrt{3}$ is irrational$\}$ be a relation on X then R is

A Reflexive and Symmetric

B Reflexive only (Ans)

C Symmetric only

D None of these

Homework

Competetive level

Let S be the set of all real numbers. Then the relation R = $\{(a, b) : 1 + ab > 0\}$ on S is:

A Reflexive and symmetric but not transitive (Ans)

B Reflexive, Symmetric and Transitive

C Reflexive and transitive but not symmetric

D None of these

THANK
YOU